pti Technical Notes

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Cracking in Ground-Supported Post-Tensioned Slabs on Expansive Soils

By Ken Bondy[1]

1 - INTRODUCTION

Post-tensioned slabs are widely used as foundations for light residential construction in areas of expansive soils. Common design methods for these slabs assume an uncracked section, where slab geometry and prestress forces are selected such that concrete flexural stresses under anticipated service loads are limited to a value less than the concrete modulus of rupture, and differential slab deflections are within acceptable limits[2]. Post-tensioned ground-supported slabs in residential applications have the following general characteristics:

- They are lightly post-tensioned with unbonded tendons. Typical average compression levels range between 50 psi (0.34 MPa) and 100 psi (0.69 MPa).
- They contain very little bonded reinforcement.
- Their cracking moment M_{cr} is normally larger than their conventional flexural strength ϕM_n, calculated with a cracked section and the internal "T-C" couple.

The successful performance of many thousands of post-tensioned ground-supported slabs built over the past several decades, combined with the fact that some cracks can be found in virtually all of them, suggests that cracking is not detrimental to their structural behavior. However the unique properties of post-tensioned ground-supported slabs stated above also suggest that the ramifications of cracking must be carefully considered. This paper intends to do just that.

2 - THE STRUCTURAL FUNCTION OF GROUND-SUPPORTED POST-TENSIONED SLABS

A ground-supported post-tensioned slab acts as a buffer which reduces differential deformations between the soil below and the superstructure above. The slab is designed with the capability of either resisting or spanning over moisture-induced deformations in the soil below, while still maintaining its top surface within permissible level tolerances. The degree of levelness required at the top slab surface is a function of the type of superstructure and its ability to resist differential deformations.

3 - BEHAVIOR OF POST-TENSIONED GROUND-SUPPORTED SLABS

Loading on ground-supported post-tensioned slabs comes from above (the superstructure loads) and below (loads generated by volume changes, swelling or shrinking, in the expansive soil). Most of the soil volume changes occur in a relatively short distance (2ft-6ft) [0.6m-1.83m] from the slab edge where the soil moisture content varies[3]. This distance is known as the edge moisture variation distance, e_m. Between the e_m distances on opposing sides of the slab (in the center region of the slab) the soil moisture content remains relatively constant and no significant volume changes occur in the soil. If the soil moisture content is higher at the slab edge and decreases from the slab edge inward, the edge of the slab will rise relative to the rest of the slab in a condition known as ***edge lift.*** If the soil moisture content is lower at the slab edge and increases from the slab edge inward, the edge of the slab will drop relative to the rest of the slab in a condition known as ***center lift*** (more appropriately called ***edge drop***).

The soil volume changes, combined with the superstructure loading, produce bending moments, shears, and differential deflections in the post-tensioned slab. The maximum moments occur at a distance from the slab edge known as the "β distance." The continuous line at a distance β from the slab edge is called the "β line." The area between the slab edge and the β line is called the "β zone." β is defined numerically (in feet or meters) as follows[2]:
where:

$$\beta = \frac{1}{12}\sqrt[4]{\frac{E_c I}{E_s}}$$

$$\beta = \frac{1}{1000}\sqrt[4]{\frac{E_c I}{E_s}}$$

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[2] *Design and Construction of Post-Tensioned Slabs-on-Ground,* Post-Tensioning Institue, 1980, Chapter 6.

[3] ibid, Figure A.3.4, p.34.

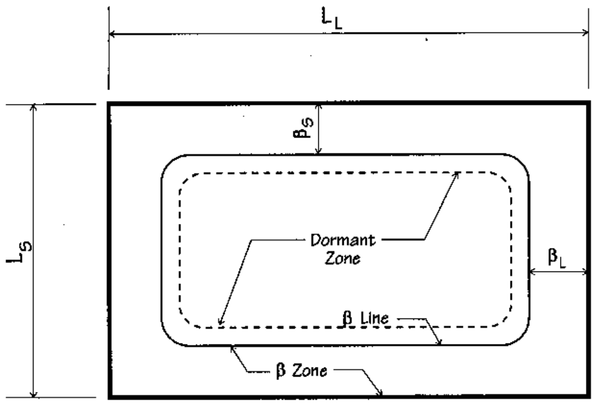
[4] ibid, Chapter 5.

E_c = Modulus of elasticity of the concrete, psi. (MPa)

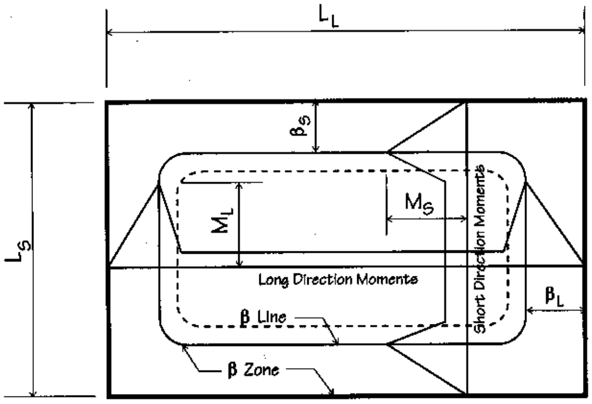
E_s = Modulus of elasticity of the soil, psi. (MPa)

I = Gross moment of inertia of the slab perpendicular to direction of bending, in^4. (mm^4)

Typical β dimensions for post-tensioned ground-supported slabs range from 6ft (1.83m) to 10ft (3m). Moment gradients are steep on either side of the β distance. The moment increases rapidly from zero at the slab edge to a maximum at the β distance, and then falls off rapidly towards the center of the slab to a relatively small value. The central portion of the slab, an area just inside the β zone, is a dormant area of small moments, shears, and differential deflections. Stated simply, everything structurally significant that happens in post-tensioned ground-supported slabs happens between the slab edge and a point just inside the β line. The β distance, the β line, the β zone, and the dormant area are shown in Figure 1. Simplified (straight-line) orthogonal moment patterns for a typical rectangular slab plan are shown in Figure 2.



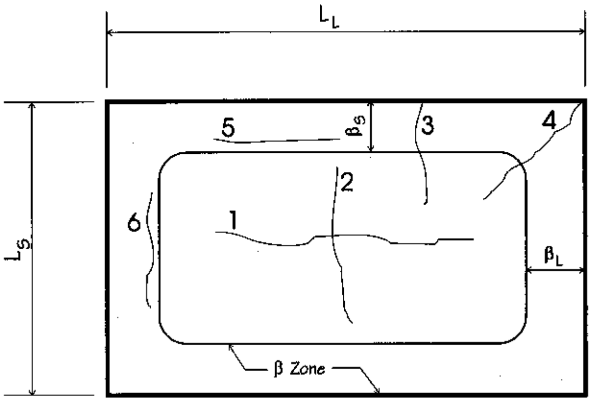
Slab Plan
Figure 1



Slab Plan
Figure 2

Since virtually all of the structural activity in post-tensioned slabs on ground occurs within the β zone, it follows that cracking outside the β zone (more precisely, in the dormant area) is of negligible structural consequence. Further, except at the slab corners, the bending mode within the β zone is primarily one-way, generating bending stresses only on concrete planes which are ***parallel*** to the slab edge. Thus it also follows that cracks which are in the β zone but ***perpendicular*** to the slab edge (or the β line) are of negligible structural concern, since they are normal to the planes upon which bending stresses act (i.e, they are ***parallel*** to the slab "span"). By a logical process of elimination, then, it can be stated that the only crack orientation in post-tensioned slabs on ground which could have potential structural consequence is a crack within the β zone and ***parallel*** to the slab edge or the β line.

Figure 3 shows a rectangular slab plan with six crack orientations. Cracks 1 and 2 are outside the β zone (in the dormant area) and are not structurally significant. Crack 3 is in the β zone but perpendicular to the β line so it too is not structurally significant. Cracks 4, 5 & 6 are the crack orientations shown which have potential structural consequence since they are within the β zone and are either parallel to the β line or have components parallel to the β line.



Slab Plan
Figure 3

4 - CATEGORIES, CAUSES, AND CHARACTERISTICS OF CRACKING

Two broad categories of cracking can occur in ground-supported post-tensioned slabs. These categories of cracks are caused by entirely different phenomena and have very different characteristics. They are:

- Cracks caused by the restraint to axial slab shortening (RTS cracks).
- Cracks caused by applied loads (flexural or shear cracks).

Each of these two categories of cracking is discussed as follows.

4.1 - RESTRAINT-TO-SHORTENING CRACKS

The most common type of cracking found in post-tensioned slabs on ground is caused by restraint-to-shortening (RTS). RTS cracking occurs when the normal tendency of a post-tensioned slab to shorten due to the effects of elastic shortening, shrinkage, and axial creep, are restrained[5]. In ground-supported slabs, most of the restraint to shortening is provided by friction between the slab and the supporting soil, and to a lesser degree, the "keying" action of grade beams (if any) embedded into the soil.

The total shortening strain ε which occurs in a typical post-tensioned slab on ground is in the range of 0.0004 to 0.0006 in/in (mm/mm). Since compression levels are low, most of this strain consists of concrete shrinkage. Assuming a longterm concrete creep modulus of elasticity E_c of 1,500,000 psi (10,345 MPa) and a midrange value for ε of 0.0005, the tensile stress which would be developed in the concrete if shortening was ***totally*** restrained is:

$$\sigma = E_c\varepsilon = 1{,}500{,}000 \times 0.0005 = 750 \text{ psi}$$
$$(10{,}345 \times 0.0005 = 5.17\text{MPa})$$

For a slab with f'_c of 2500 psi (17.24MPa), the concrete modulus of rupture is $7.5\sqrt{2500} = 375$ psi ($0.625\sqrt{17.24} = 2.6$MPa). Assuming an average prestress compression stress of 100 psi (0.69 MPa) on the concrete, it would require an applied tensile stress of 375+100 = 475 psi (2.6+0.69 = 2.29MPa) to crack the slab. Thus only about 63% of full restraint (475/750) [2.29/5.17] is required to produce a restraint-to-shortening crack in a post-tensioned slab on ground. This demonstrates why most post-tensioned slabs on ground contain some RTS cracks.

RTS cracks have the following general characteristics, which can be helpful in their diagnosis:

- They occur shortly after the concrete is placed, often before the tendons are stressed.
- They are independent of applied load, and occur in locations which have no direct relationship to points of maximum slab moment and shear.
- They are substantially vertical and penetrate the entire depth of the concrete slab.

The mechanics of RTS crack formation are important in evaluating the ramifications of RTS cracking in post-tensioned slabs on ground. Figure 4 shows a freebody of a post-tensioned slab on ground of length L and with the tendons removed and replaced with the forces F_o that they apply at the slab edges. The slab tries to shorten towards its geometric center at $L/2$ (every point in the slab tries to move towards point A) and that movement is resisted by frictional forces between the soil and the slab acting at the bottom of the slab. The frictional forces are expressed in terms of the resistance per unit of slab length f.

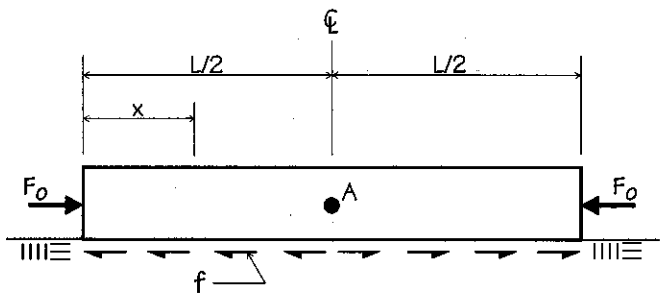


Figure 4

Figure 5 shows a freebody of a portion of the slab of length x where x<*L*/2. The axial force acting on the concrete surface at x is:

$$F_x = F_o - fx$$

If $F_o > fx$ then F_x acts in the direction shown in Figure 5 and the slab is in ***compression*** at x. If $F_o < fx$, however, the direction of F_x reverses and the slab is in ***tension*** at x, and the tension increases with increasing x, until x reaches *L*/2. At some value of x the tensile force F_x may reach F_r, the force required to fail the concrete in tension, and an RTS crack will be produced at that point.

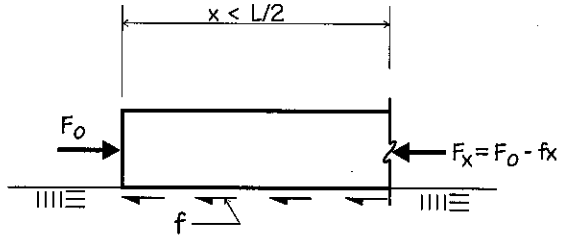


Figure 5

The maximum tensile value of F_x occurs when x = *L*/2 (the direction of the friction force reverses there). In a slab with little bonded reinforcing to distribute cracks, the first RTS crack, if it occurs, will therefore typically occur near the geometric center of the slab. Once this first crack has developed, the slab appears as in Figure 6a, which shows the incremental friction forces which develop after the crack appears. The crack has now separated the slab into two pieces, each of length *L*/2, with each piece now shortening towards its own individual geometric center at points B and C. If the tensile force F_x acting at B and C reaches F_r, another RTS crack will develop near B and C and the slab will then look like Figure 6b with ***four pieces*** now each shortening towards their individual geometric centers.

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[5] Aalami, Bijan and Barth, Florian, "Restraint Cracks and their Mitigation in Unbonded Post-Tensioned Building Structures", Post-Tensioning Institute, Phoenix, Arizona pp 49, 1988.

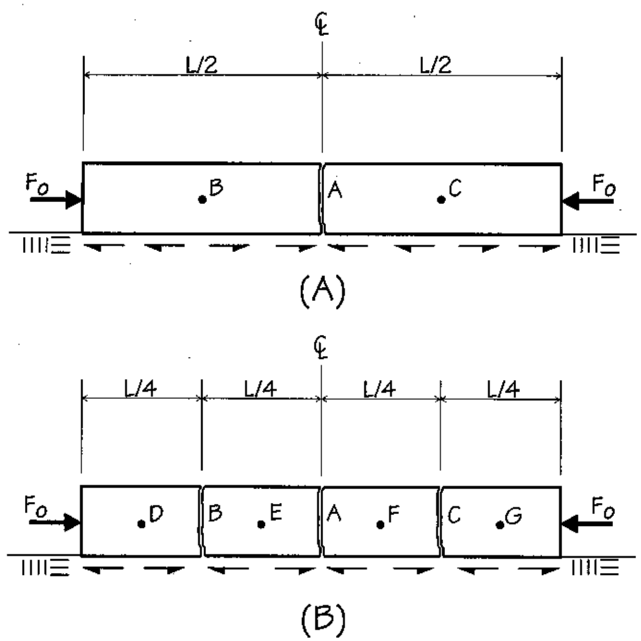

Figure 6

This theory of RTS crack development is consistent with the author's observations of many post-tensioned slabs on ground. Obviously RTS cracking is influenced by many secondary factors such as slab shape, slab openings, re-entrant corners, etc., nonetheless predominant observed locations for RTS cracks are at or near slab centers and quarterpoints. For unusually long slabs, and slabs with exceptionally high shortening strains (for example, concrete with high water/cement ratios), cracks can occasionally be found at or near eighth points along the slab length. It can be seen that the majority of RTS cracking occurs within the center half of the slab, and RTS cracks which penetrate the β zone do so at right angles to the β line. In many years of observing existing post-tensioned slabs on ground, the author has never seen an RTS crack which is inside the β zone ***and*** parallel to the slab edge.

4.1.1 - RAMIFICATIONS OF RTS CRACKING

It is concluded, then, that because of their normal orientation, RTS cracks are of no structural consequence in post-tensioned slabs on ground. This conclusion is consistent with the actual performance of existing post-tensioned slabs on ground, many of which have significant RTS cracking and are performing in a completely functional manner.

4.2 - APPLIED LOAD CRACKS

When applied loads produce concrete stresses which exceed the concrete modulus of rupture, a crack is produced. If the crack is caused by direct bending stresses acting on the concrete cross-section, it is called a flexural crack. If the crack is caused by diagonal tension stresses generated by shear, it is called a shear crack. This discussion will focus on flexural cracking, however the same conclusions can be drawn with respect to shear cracking.

Most post-tensioned slabs on ground are designed to limit flexural stresses to $6\sqrt{f'_c}$ $(0.5\sqrt{f'_c})$. Since concrete fails in tension at a stress which is normally assumed to be $7.5\sqrt{f'_c}$ $(0.625\sqrt{f'_c})$ (the modulus of rupture), flexural cracking does not appear until the applied loads exceed design loads by about 25%. The study of applied load crack behavior in post-tensioned slabs on ground, therefore, deals primarily with their capability to sustain ***overloads,*** loads substantially in excess of anticipated design loads. This study will also address a means for evaluating the geometric functionality of the slab (see Section 2) in the overloaded condition.

Flexural cracks have the following characteristics:

- They are located at points of maximum slab moments (at or very near the β line).
- They do not penetrate the full depth of the slab. They can be seen only on the slab surface (top or bottom) where tension is produced by applied loads.
- They are independent of the age of the concrete slab. They are only a function of applied loads.

Slab behavior is elastic prior to the appearance of the first crack. This means that there is a linear relationship between applied loads (or moments) and differential slab deflections. Post-cracking behavior is a function of whether the slab is deforming in a center lift mode or an edge lift mode. The differences in behavior are discussed below:

4.2.1 - POST-CRACKING CENTER LIFT BEHAVIOR

In the center lift mode, the soil in the edge moisture variation distance e_m shrinks away from the slab. This reduces the upward soil loading on the slab near the slab edge, and causes a small (theoretical) gap between the slab and the soil extending approximately over the e_m distance[6]. The slab, in the uncracked condition, acts very much like a free cantilever with a length of β as shown in Figure 7. In the center lift mode, slab moments are negative (tension at the top of the slab) and they are increased by increasing the superstructure loads.

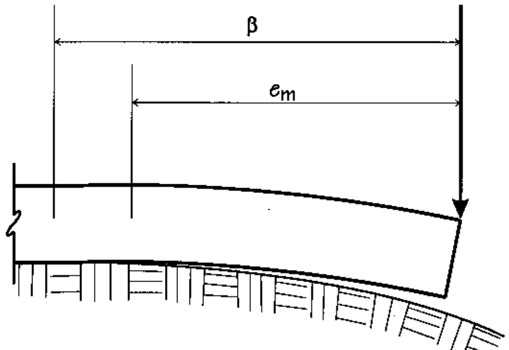

Figure 7

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[6] Design and Construction of Post-Tensioned Slabs-on-Ground, Post-Tensioning Institute, 1980, Figure 4.2, p.8.

If soil support is ignored slab behavior would be as shown in the load-deflection curve of Figure 8. In this load-deflection curve, and in the ones which follow, the deflection is assumed to increase from zero to some maximum value and the applied loads P are those which are required to sustain the specific deflections. In Figure 8 deflection and load increase elastically until at an applied load of P_{cr}, (the load required to produce first cracking), a small additional deflection results in a drop in the slab resistance and increased rotation at the crack point. The load drops to P_n, which is statically consistent with the conventional cracked section moment strength ϕM_n. ϕM_n is developed by an internal couple between the tensile force in the tendons T and the compressive force in the concrete C. The slab then acts as a "hinge" at the point of the crack, following the applied displacement with no increase in resisting moment until the concrete fails in compression.

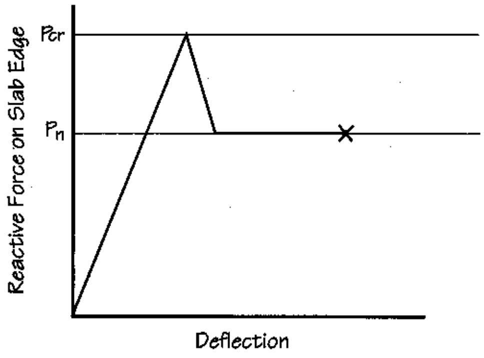

Figure 8

In a ground-supported slab, however, the full rotation required after first cracking to "unload" the slab to the load consistent with its nominal strength P_n is likely to be prevented by the soil in the β zone. After cracking, the slab rotates at the crack point. With incremental deflection the soffit contacts the soil and begins to resist the deflection. Consistent with the increased deflection and reduced slab stiffness the reactive load drops to a value between P_{cr} and P_n,. Further downward displacement is balanced by the combined actions of the soil and the slab (see Figure 9). The load-carrying capacity of the slab/soil system after cracking can be equal to and even larger than the cracking load P_{cr}.

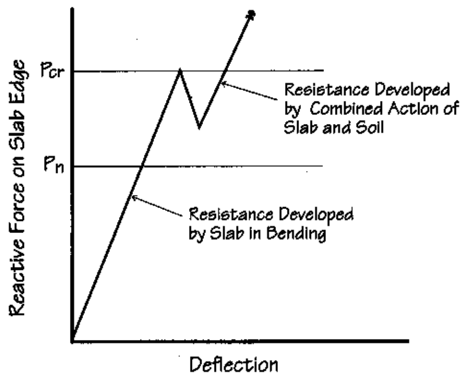

Figure 9

4.2.2 - POST-CRACKING EDGE LIFT BEHAVIOR

In the edge lift mode, ***the soil exerts upward loads on the slab edge***[7], slab moments are positive (tension at the bottom of the slab), and the moments increase with increases in soil loading (increased swelling in the e_m distance). Increases in superstructure loading ***decrease*** the slab moments in the edge lift condition. Figure 10 shows the deformed slab shape in the edge lift mode.

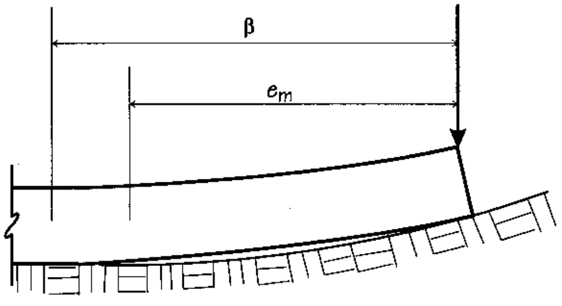

Figure 10

When the first crack appears at the slab soffit at the β line, at an applied upward soil load of P_{cr}, the slab tries to rotate in the direction of applied soil loading (i.e., up and away from the soil). There is no significant deflection or rotation which occurs immediately after cracking as in the center lift mode, where the slab is not in contact with the soil at the time of cracking. Increased upward deflection after cracking is accompanied by soil loading beyond P_{cr} to account for loading from the superstructure. Eventually the slab concrete fails in compression at a load substantially larger than P_{cr}. The slope of the post-cracking load-deflection curve is less than the uncracked slope, explained by the reduction in slab stiffness at cracking. Edge lift behavior is shown in Figure 11.

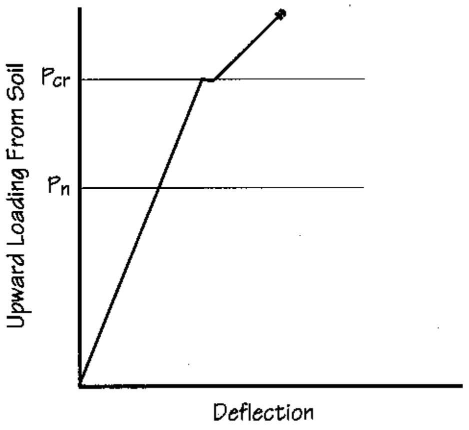

Figure 11

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[7] Design and Construction of Post-Tensioned Slabs-on-Ground, Post-Tensioning Institute, 1980, Figure 4.2, p.8.

As in the center lift mode, the slab bending in an edge lift mode can sustain loads in excess of P_{cr} after cracking because of increased post-cracking soil support and loading.

4.2.3 - RAMIFICATIONS OF APPLIED LOAD CRACKING

Flexural cracking caused by applied loads has no significant effect on the ***strength*** of ground-supported post-tensioned slabs. This is true because of the increase in soil support which is mobilized after cracking occurs in the slab. The soil, in effect, provides the factor of safety which is required by code strength provisions.

However, applied load β-zone cracking can cause excessive slab deformations which may affect the ***serviceability*** of the slab. Ground-supported post-tensioned slabs are designed to partially separate from the soil below them, either by spanning the separation or cantilevering over it. In doing this the slab must maintain acceptable deformation limits while carrying the superstructure loads from above. The slab is generally designed to accomplish this span in an uncracked state. If the slab cracks, the stiffness is reduced and the deformations are amplified.

Post-cracking deformations are cumbersome to compute. They are further complicated by the imprecise nature of information describing the condition of the existing slab, which makes a rigorous analytical evaluation unreliable. Thus, a slab which exhibits true applied load β-zone cracking must be evaluated on an individual qualitative basis by an experienced structural engineer.

The condition of the structure above the slab (distress specifically related to soil movement such as drywall or plaster cracking, door and window racking, etc.) along with the levelness of the top of the slab (with due consideration to realistic construction tolerances) provide the primary data with which the structural engineer can evaluate the performance of the slab. Other factors affecting the evaluation include the age of the slab and its past history of changes in underlying soil moisture and how those changes compare to those assumed in its original design. If all of these data are determined to be within reasonable subjective limits, the slab can be considered to be functional. If not, remedial measures may be required.

This description of post-cracking behavior, and the conclusions reached regarding its significance, are supported by published mathematical studies[8] and by the observable behavior of post-tensioned ground-supported slabs.

5 - SUMMARY AND CONCLUSIONS

This Technical Note has addressed the following:

- The general behavior of post-tensioned slabs-on-ground.
- The categories and characteristics of cracking (restraint-to-shortening cracks and applied loading flexural cracks) which occur in post-tensioned slabs-on-ground and their ramifications.

It is concluded that most cracks in post-tensioned slabs on ground are not detrimental to their structural function. Restraint-to-shortening (shrinkage) cracks, because of their typical orientation within the slab dormant zone, have no effect on slab strength or differential slab deflections. Flexural cracking has no effect on the strength of ground-supported slabs because of the soil support available after cracking occurs. Flexural cracking, however, can have a detrimental effect on slab serviceability by increasing post-cracking deformations. Slabs which exhibit applied load flexural cracking must be evaluated on an individual basis by a structural engineer experienced in post-tensioned ground-supported slabs.

These conclusions are consistent with the observed behavior of many thousands of post-tensioned slabs on ground built since the late 1960's, most of which have some level of visible cracking and are functioning in a satisfactory manner.

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[8] *Design and Construction of Post-Tensioned Slabs-on-Ground,* Post-Tensioning Institute, 1980, Figure 5.2(F) and Figure 5.4, pp.11-12.

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