Variable Prestress Force in Unbonded Post-Tensioned Members

by Kenneth B. Bondy

Design methods for unbonded post-tensioned concrete flexural members have historically assumed that the effective prestress force is constant for the full length of the tendon. Designers theorized that the variable tendon force that exists due to friction at the time of stressing distributes with time until the force becomes equal at all points in the tendon. Consistent with this assumption, effective tendon forces are normally based on the average initial force in the tendon minus the long-term losses. Thousands of successful post-tensioned concrete structures have been designed over the past thirty-plus years using this constant force approach.

However, a study published in 1991[1] concludes that the assumed redistribution of tendon force does not occur and that the design of unbonded post-tensioned members should be based on a variable effective prestress force. The variable force is determined by subtracting long-term losses from initial tendon forces acting at each point along the tendon profile. It is possible that a variable force approach will eventually be the recommended industry standard for designing unbonded post-tensioned building members.

There are major mathematical complications inherent in a variable force approach. Tendon "balanced loads," flexural concrete stresses, unstressed flexural and shear reinforcement, and secondary moments are all functions of the prestress force and calculating each of these becomes much more complex if the force is variable. The impact of a variable force approach on designing post-tensioned concrete building members is largely unknown. The satisfactory performance of the many members designed with constant force suggests that the consequences are negligible. But that is speculative, since there is virtually no data available on members designed with variable force. Until recently, computer programs for designing post-tensioned beams and slabs have been based on constant prestress force, and a variable force analysis for even simple indeterminate post-tensioned members is more complex than most engineers would attempt by hand.

PTDATA+, a computer program developed for designing and analyzing post-tensioned concrete members with variable prestress force, was used to compare typical post-tensioned concrete beams and slabs designed with constant and variable prestress forces.

Theory

Immediately after anchoring, the force distribution in a tendon stressed from one end is shown by Curve A of Fig. 1. The force in the tendon can be calculated at any

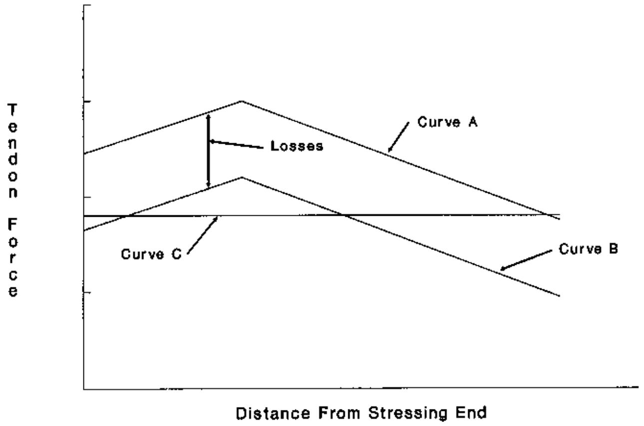

Fig. 1 — Tendon force profile.

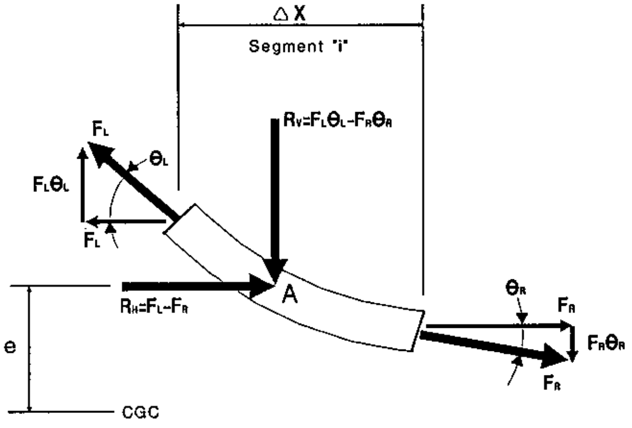

Fig. 2 — Forces acting on tendon segment "i."

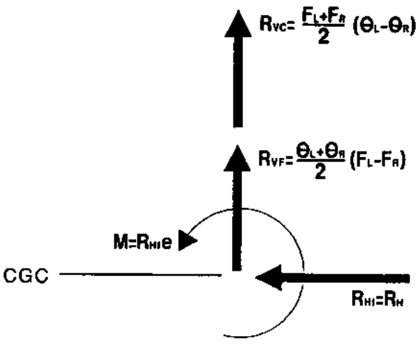

Fig. 3 — Equivalent concrete loads at segment "i."

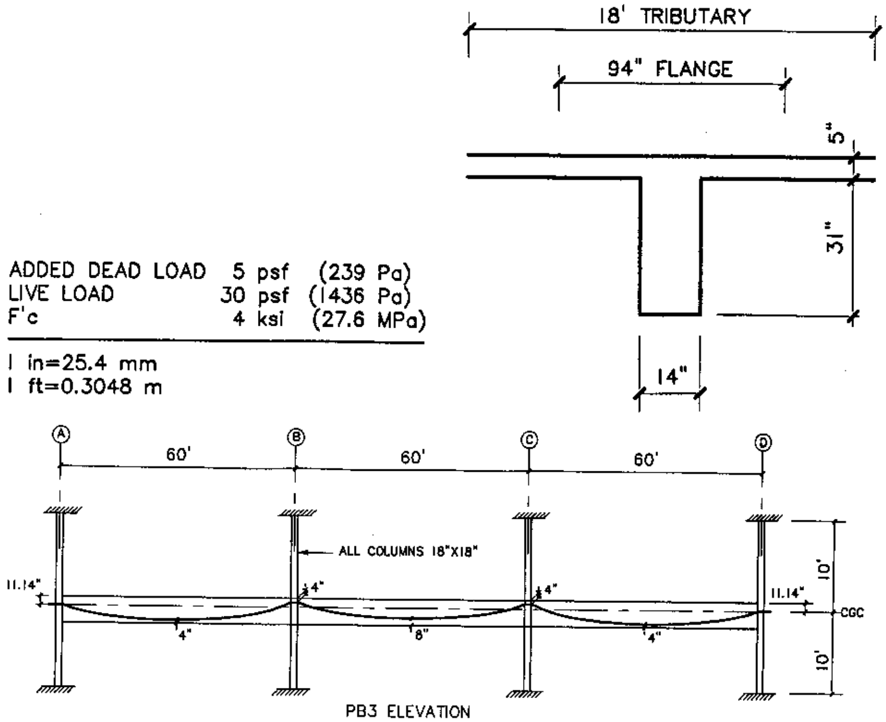

Fig. 4 — PB3 details.

point along its length using ACI 318-89,[2] Eq. (18-1), with due consideration for the effect of an anchor set near the stressing end. For practical purposes, elastic shortening and long-term tendon stress losses (shrinkage, creep, and steel relaxation) may be considered constant throughout the tendon length. If no redistribution occurs, the effective tendon force profile is represented by the variable force Curve B, which is obtained by subtracting a constant long-term loss value from each point on Curve A. In the constant force approach, the tendon force is assumed to redistribute after anchoring until (after long-term losses) the force is constant, as shown by Curve C. The area under curves B and C are equal, each representing the total energy stored in the tendon after all losses.

A small segment of the tendon is isolated as a free body in Fig. 2. In this elemental segment, the tendon is curved (the tendon slope is different at each end of the segment), and the force is variable (the tendon force is different at each end of the segment). The concrete exerts a force on the tendon segment to equilibrate it and this force is shown in vertical and horizontal components R_V and R_H in Fig. 2. Angles θ_L and θ_R are assumed to be small, so that the horizontal components of the tendon force F_L and F_R can be taken to be equal to F_L and F_R, respectively.

The vertical component R_V of the force exerted on the concrete by the tendon segment can be broken down into two parts, as shown in Fig. 3. The first part R_{VC} is produced by tendon curvature and is equal to the average force in the segment multiplied by the difference in the tendon slope at each end of the segment. The second part R_{VF} is produced by tendon friction and is equal to the average slope of the tendon segment multiplied by the difference in tendon force at each end of the segment. The effects of tendon friction on the segment R_{HF} and R_{VF} can thus be isolated from the effects of tendon curvature R_{VC}. Note that the forces R_{HF}, R_{VF}, and R_{VC} are shown in Fig. 2 acting on the tendon segment. Fig. 3 shows the equal and opposite set of forces acting on the concrete at each tendon segment.

In PTDATA+, the "load balancing" or equivalent load method of analysis is used for calculating elastic effects in the indeterminate post-tensioned member. In the equivalent load method, the tendon is replaced with the loads it exerts on the concrete, and the concrete alone as a free body is then analyzed for the effects of these loads. To model the effects of the tendon loads with variable prestress force, each span is divided into ten segments and the tendon loads R_{VC}, R_{VF}, and R_{HF} are calculated for each segment, as shown in Fig. 3.

The horizontal friction component R_{HF} produces the variable tendon force and an applied concentrated moment $R_{HF} \cdot e$, assumed to act at the center of the segment, where e is the distance between the tendon center of gravity of steel (CGS) and the center of gravity of the concrete (CGC) cross section. The vertical component of the tendon friction force R_{VF} is assumed to act as a point load at the center of the segment. The curvature load R_{VC} is uniformly distributed over the segment if the tendon profile is parabolic. Sharp bends in the tendon profile produce point loads on the concrete at the bend points, and a change in member cross section produces a concentrated moment at that point.

Comparative study

The three-span post-tensioned Beam PB3 shown in Fig. 4 was selected to evaluate the effects of variable tendon force. This is a commonly used application of post-tensioned con-

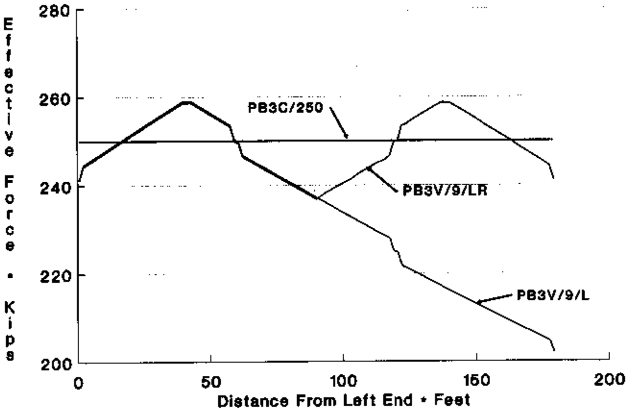

Fig. 5 — Total effective prestress force.

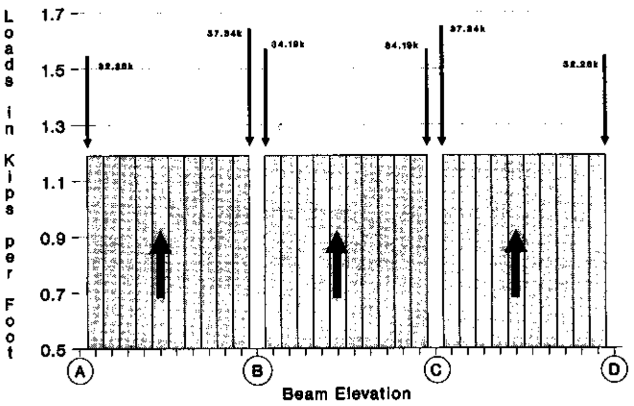

Fig. 6 — Equivalent loads — curvature R_{vc} for PB3C/250.

crete and, because of its length and depth, one that was felt to be particularly sensitive to a variable force.

The beam was first designed using a conventional constant force method. This design was designated as PB3C/250. The resulting beam was then analyzed twice using a variable force method, first assuming tendon stressing from both ends (designated PB3V/9/LR) and then assuming tendon stressing from only one end (designated PB3V/9/L). Specific parameters studied for comparison in the three beams were tendon force, equivalent tendon loads, concrete bending stresses, and ultimate strength reinforcing steel requirements. All references herein to "Code" or code sections refer to ACI 318-89.[2]

The constant force design PB3C/250 conformed to the following criteria:

- Tendons are continuous from end to end of the beam (no added tendons)
- Allowable concrete stresses (f'_c = 4 ksi [27.6 MPa]):

  Tension $6\sqrt{f'_c}$ = +0.379 ksi (2.61 MPa)

  Compression $0.45f'_c$ = −1.800 ksi (−12.41 MPa)
- Tendon profiles are parabolic between support faces and straight across the support (a commonly used balance-to-face profile modeling assumption).

This design requires an effective prestress force of 250 kip (1112 kN), controlled by the maximum allowable flexural tensile stress of 0.379 ksi (2.61 MPa) at the bottom beam fiber in the two exterior spans (the actual calculated stress there is 0.376 ksi [2.59 MPa]). All other stresses are within allowable values. Tendon equivalent loads, concrete bending stresses, and reinforcing steel requirements for PB3C/250 are shown in Fig. 6, 13, and 16, respectively.

In a variable force analysis the actual cross-sectional area of prestressing steel (or the number of ten-

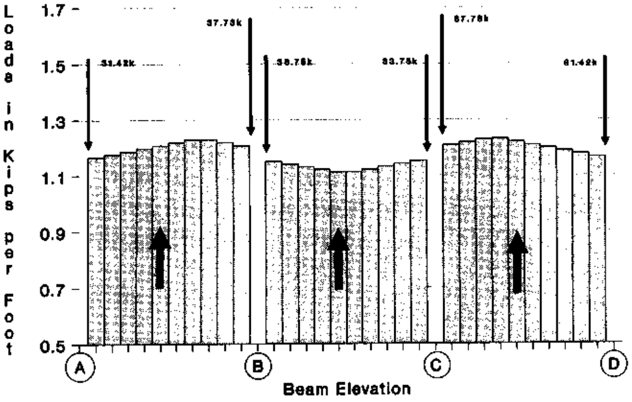

Fig. 7 — Equivalent loads — curvature R_{vc} for PB3V/9/LR.

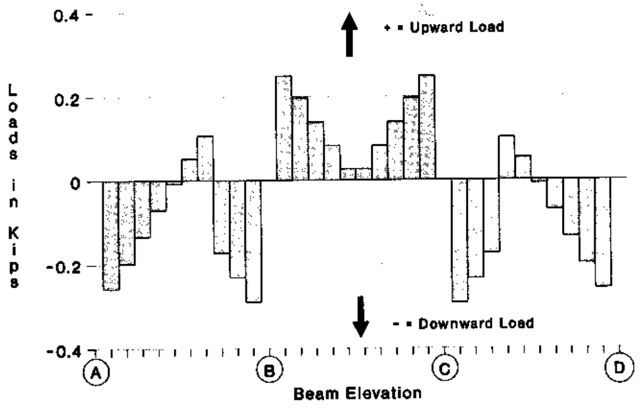

Fig. 8 — Equivalent loads — friction R_{vF} for PB3V/9/LR.

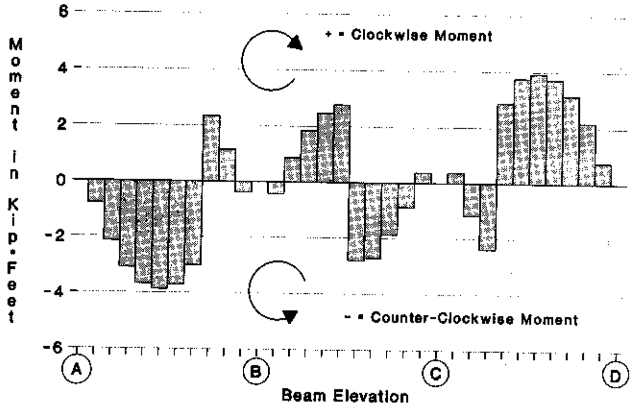

Fig. 9 — Equivalent loads — friction $R_{HF} \cdot e$ for PB3V/9/LR.

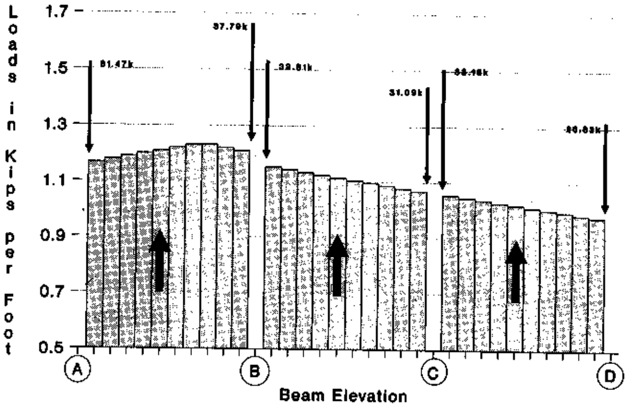

Fig. 10 — Equivalent loads — curvature R_{VC} for PB3V/9/L.

dons with known properties) must be determined. For typical ½ in. (13 mm) diameter, 270 ksi (1860 MPa), low-relaxation tendons, average effective forces in the range of 26.0 to 27.5 kips (115.7 to 122.3 kN) each are commonly assumed. Using the low end of this range would require 250/26 = 9.6 tendons, while the high end would require 250/27.5 = 9.1 tendons. Therefore, most designers would specify either 9 or 10 tendons for this beam. In this study, 9 tendons were selected for analyses using the variable force approach.

Tendon stressing points must be known or assumed and specified as a part of the variable force analysis criteria. In this study, two tendon stressing patterns were analyzed. In PB3V/9/LR, the tendons were assumed to be stressed from both the left and right ends of the beam (two-end pulls). In PB3V/9/L, the tendons were assumed to be stressed only from the left end of the beam (one-end pulls). Also, since friction and long-term losses are integral parts of the variable force method, data defining those factors must be known. For the variable force analyses, the additional data were as follows:

- Friction wobble coefficient k = 0.001
- Friction curvature coefficient μ = 0.07
- Concrete strength at stressing f'_{ci} = 3 ksi (20.7 MPa)
- Age of concrete at time of tendon stressing — 5 days
- Average ratio of initial to ultimate tendon stress — 0.7
- Relative humidity — 60 percent
- Tendons — ASTM A 416, ½ in. (13 mm), 270 ksi (1860 MPa), low relaxation
- Anchor set — ¼ in. (6.3 mm)
- Modulus of elasticity of tendons — E_s = 28,000 ksi (193,000 MPa)

Using the stated criteria, the resulting effective tendon force diagrams for all three beams are shown in Fig. 5. Tendon equivalent loads resulting from the variable prestress force are shown in Fig. 7, 8, and 9 for PB3V/9/LR and Fig. 10, 11, and 12 for PB3V/9/L. Fig. 7 and 10 show the curvature loads (the familiar balanced loads) normal to the tendon profile. Fig. 8, 9, 11, and 12 show the loads produced by tendon friction — loads acting on the concrete parallel to the tendon profile. The vertical component of these friction loads R_{VF} is shown in Fig. 8 and 11. The applied moments produced by the horizontal component R_{HF} are shown in Fig. 9 and 12.

The results of the variable force analysis for concrete bending

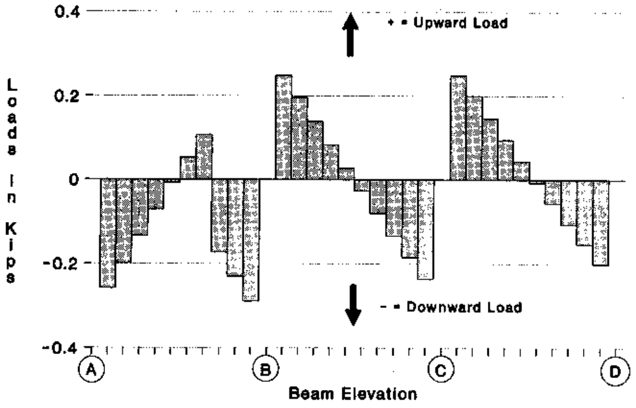

Fig. 11 — Equivalent loads — friction R_{VF} for PB3V/9/L.

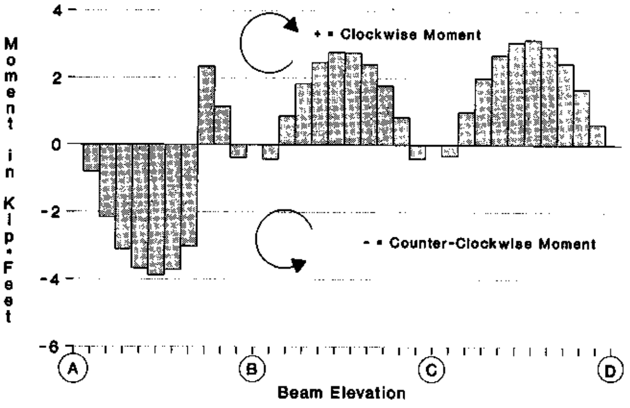

Fig. 12 — Equivalent loads — friction $R_{HF} \cdot e$ for PB3V/9/L.

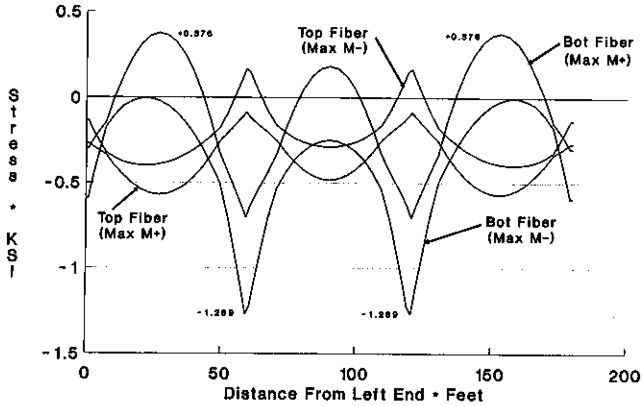

Fig. 13 — Concrete bending stresses for PB3C/250.

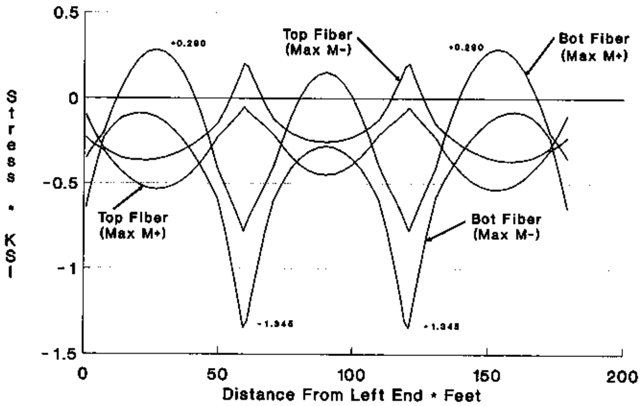

Fig. 14 — Concrete bending stresses for PB3V/9/LR.

stresses and unstressed reinforcing steel are shown in Fig. 14 and 17, respectively, for PB3V/9/LR, and in Fig. 15 and 18, respectively, for PB3V/9/L. The reinforcing steel requirement calculated in Fig. 16 through 18 is under the action of factored dead and live loads 1.4DL + 1.7LL using both tendons and reinforcing steel for tensile resistance.

In the concrete bending stress diagrams, Fig. 13 to 15, stresses are plotted at the top and bottom beam fibers. The stresses are caused by the most positive and most negative moments that can occur at those fibers (as a result of skipped live load). For example, the curve labeled "Bot Fiber (Max M+)" shows the stress at the bottom fiber produced by the maximum possible positive moment at that point. Thus, the four curves show the entire range of stresses possible at any point along the beam.

Other parameters, such as equivalent load moments, secondary moments, and deflections, could, of course, be extracted from the analyses for the beams with variable force and compared with PB3C/250. However, concrete bending stresses and reinforcing steel requirements are items directly limited by code and are felt by the author to be most pertinent for comparison.

Concrete bending stresses

The two-end pull beam PB3V/9/LR has substantially lower concrete tensile stresses and slightly higher compressive stresses than PB3C/250. The controlling tensile stress was reduced from 0.376 ksi (2.59 MPa) in PB3/250 to 0.290 ksi (2.0 MPa) in PB3V/9/LR, a 23 percent reduction. This is because of the generally higher effective prestress force in the exterior spans. The compressive stresses in the bottom beam fibers at the two interior supports are increased in PB3V/9/LR by about 6 percent, but the increased stress of −1.345 ksi (9.27 MPa) is still well below the allowable compressive stress of −1.800 ksi (12.4 MPa).

In the one-end pull design PB3V/9/L the tensile stress in the right end span increases to 0.479 ksi (3.3 MPa). This exceeds the code allowable stress of $6\sqrt{f'_c}$ in Section 18.4.2(b) but is less than the $12\sqrt{f'_c}$ permitted in Section 18.4.2(c) with additional deflection and cover restrictions. Thus, the variable force in this beam would change the code category for allowable tensile stresses [from Section 18.4.2(b) to Section 18.4.2(c)] and require additional considerations of deflection, tendon cover, and reinforcing steel cover for compliance (see Section 7.7.3.2). An examination of the PTDATA+ analysis indicates that the beam, in fact, does satisfy the additional requirements of Section 18.4.2(c) and therefore is code-compliant as to flexural tensile stresses, although it does exceed the limits of Section 18.4.2(b). Compressive stresses in PB3V/9/L increase over those in PB3C/250 to a maximum of −1.426 ksi (−9.83 MPa) at grid line "C" which is still well within the allowable maximum value of −1.800 ksi (−12.4 MPa).

Reinforcing steel requirements

Grade 60 reinforcing steel requirements for flexural strength under 1.4DL + 1.7LL in PB3V/9/LR increased 12 percent for top bars at the two interior supports and decreased 20 percent for bottom bars in the two exterior spans, as compared to PB3C/250. The top bar increase involved only 0.265 in.^{2} (171 mm^2) of steel, and in many cases this small increase would not result in any change in practice. For example, many designers would select two #10 bars to satisfy the requirement of 2.208 in.^{2} (1424 mm^2) in PB3C/250. Those bars would also satisfy the 2.473 in.^{2} (1595 mm^2) requirement in PB3V/9/LR.

In the one-end pull beam PB3V/9/L, the increases in reinforcing steel become rather significant as the distance from the stressing end increases. At the far interior support "C," the increase in top bars required is 40 percent, from 2.208 to 3.1 in.^{2} (1424 to 2000 mm^2). The bottom bar increase in the far exterior span "C-D" is 47 percent, from 0.786 to 1.156 in.^{2} (507 to 746 mm^2). Even though these percentage increases seem large, in practice they would involve an increase of just one bar size and may be economically via-

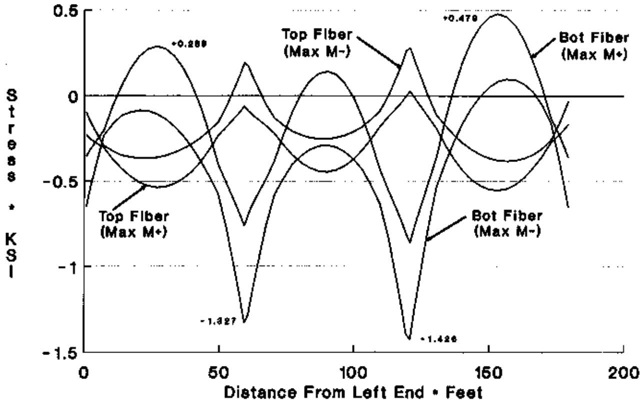

Fig. 15 — Concrete bending stresses for PB3V/9/L.

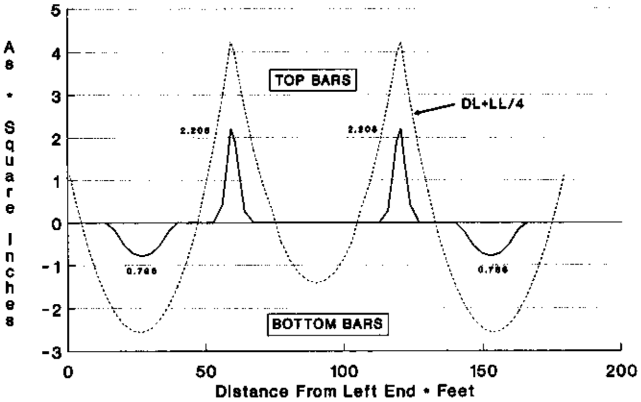

Fig. 16 — Reinforcing steel requirements for PB3C/250.

ble when compared to the labor savings realized in eliminating half the required stressing.

Note that Section 18.9.2 of the Code requires a minimum area of bonded reinforcing of 2.22 in.^{2} (1432 mm^2) in negative moment areas and 1.39 in.^{2} (897 mm^2) in positive moment areas. This minimum requirement would control in all three beams for bottom bars and in PB3C/250 for top bars over the interior supports; however, strength reinforcing steel requirements would control the top bars at the interior supports in both variable force beams.

For designs governed by the Uniform Building Code,[3] the beam must contain sufficient bonded reinforcing to support the unfactored dead load plus 25 percent of the unreduced live load independent of the tendons and using a ϕ factor of 1.0 (Section 2618(j)2B). The reinforcing steel required to satisfy this criteria is shown in the dashed line in Fig. 16 to 18. It can be seen that at every location this requirement would control the reinforcing in all three beams.

Conclusions

Obviously it is impossible to extend the results of this one-beam study to all configurations of post-tensioned concrete members; however, they do provide some insight on the general impact that a variable force design method will have.

Structural behavior

Design by variable force as compared to constant force will have negligible impact on the actual behavior of typical post-tensioned concrete structures with "normal" tendon stressing lengths. These normal stressing lengths include maximum one-end pulls of around 100 ft (30.5 m) and maximum two-end pulls of 200 ft (61 m). For example, assume a post-tensioned parking structure beam designed and reinforced by a constant force method is adjacent to a beam designed with a variable force method. The two beams would exhibit no significant difference in cracking and deflection behavior or in the factor of safety against overload.

The changes in behavior resulting from variable and constant force designs appear to be less significant than the fact that full design loads are rarely approached in most building occupancies. This probably explains the generally successful behavior of the post-tensioned concrete members designed in the past with a constant force method.

Structural analysis

In analyzing an existing post-tensioned member, the two methods could make the difference between code compliance and non-compliance. Depending on how it is

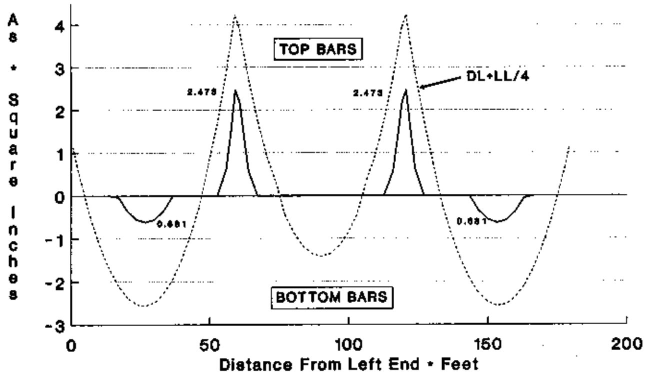

Fig. 17 — Reinforcing steel requirements for PB3V/9/LR.

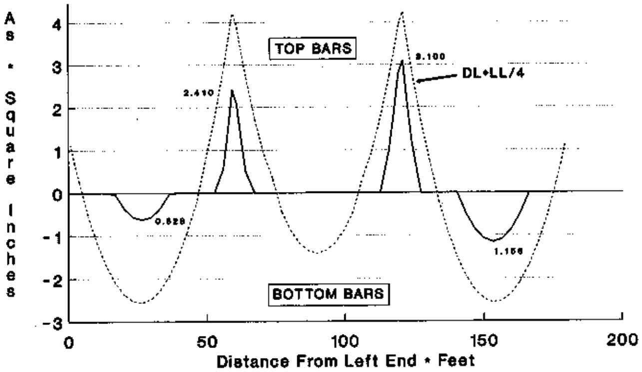

Fig. 18 — Reinforcing steel requirements for PB3V/9/L.

stressed, a beam tightly designed with a constant force method may be in violation of the code using a variable force approach. Therefore, variable force analysis could have a major impact in the field of forensic engineering. If a post-tensioned building were involved in litigation and its structural integrity questioned, evaluating the members would become a function of the analysis method used. The variable force method could, in this case, be used to exaggerate any deficiencies that might exist.

If the variable force method becomes an industry standard, the profession must address and resolve the inevitable conflict that will arise in analyzing existing members that were not designed by the variable force method.

Structural design

On a positive note, using the variable force design approach will open up areas of efficient, economical designs heretofore unavailable to post-tensioned concrete designers. For example, the 180 ft (55 m) length of one-end stressing in PB3V/9/L would be considered unacceptable by most designers whose experience has been solely with constant force approaches. In fact, this study shows that one-end stressing is quite acceptable for this beam with a change in allowable stress from $6\sqrt{f'_c}$ to $12\sqrt{f'_c}$ and with only a nominal increase in strength reinforcing steel. No increase in steel would be required under the Uniform Building Code.

Alternatively, with more conventional two-end stressing, it seems feasible that PB3V/9/LR could satisfy all code requirements with only 8 tendons (an 11 percent reduction) and a nominal increase in reinforcing steel over PB3C/250. Again, no increase would be necessary under the Uniform Building Code. Increased knowledge of the actual behavior of the member under variable force could lead to more economical designs for many types of post-tensioned concrete members.

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